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Engineering Notes

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Proportional Navigation with Adaptive Terminal Guidance for Aircraft Rendezvous

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Nomenclature

A_f	=	acceleration command at rendezvous
R_{TR}	=	distance from receiver to tanker
s	=	distance from receiver to target
V_c	=	closing velocity
V_f	=	final velocity constraint
V_R	=	receiver velocity, airspeed
V_{Rg}	=	receiver velocity, groundspeed
V_T	=	tanker velocity, airspeed
V_{Tg}	=	tanker velocity, groundspeed
x_R	=	receiver east position
x_{RZ}	=	rendezvous east position
x_T	=	tanker east position
y_R	=	receiver north position
y_{RZ}	=	rendezvous north position
y_T	=	tanker north position
Δy_{RFE}	=	distance from receiver to tanker in downtrack direction
Φ_f	=	final line-of-sight constraint
Ψ_f	=	final heading constraint
ψ_R	=	receiver heading
ψ_{RZ}	=	desired heading at rendezvous
ψ_T	=	tanker heading
$\dot{}$	=	derivative with respect to time

I. Introduction

TODAY'S manned aircraft rendezvous are facilitated by systems such as the airborne tactical air navigation system or air-to-air radar. However, for unmanned air vehicles (UAVs), a system that is capable of both rendezvous and formation flight is desired to increase precision and reduce the total number of onboard systems. A near-term solution to unmanned aircraft rendezvous and formation flight is a precision differential Global Positioning System (GPS) [1]. The difficulty in autonomous trajectory generation and guidance is the ability to accurately satisfy space and time constraints and alleviate the risks of airborne collisions and fuel shortages.

Optimal trajectory generation with dynamic optimization is one candidate method currently being applied to UAV route planning and rendezvous. Dynamic optimization techniques have been used to solve for suitable trajectories in 2-D or 3-D space while avoiding threats and obstacles [2,3]. Unfortunately, optimization algorithms are computationally intensive and lack convergence guarantees, which precludes the use of such algorithms in flight-critical UAV control systems. Another candidate method is based on a Dubins path [4] and forms a trajectory solution with multiple geometric combinations of constant-radius turns and straight lines. However, these solutions must still be iterated to meet time constraints and limit UAV maneuvering. An algorithm is desired that will compute guidance commands for a UAV-to-tanker rendezvous and will obey performance limitations of the host-aircraft, and it must be implemented efficiently into the UAV flight control system for real-time operation. The algorithm should also be robust to tanker maneuvering and arbitrary winds.

Proportional navigation (PN) guidance is well established in two-body intercept problems. It is easily implemented and is not computationally burdensome on the host flight control computer. Terminal guidance has been used in PN while assuming a constant interceptor velocity [5] and, conversely, PN has been used to intercept stationary targets by an interceptor moving at a varying velocity profile [6]. Combining terminal guidance and time-varying velocity, an adaptive proportional navigation guidance approach has been shown to be effective at guiding a lifting vehicle during its terminal phase [7]. Thus, proportional navigation, augmented with adaptive terminal guidance, is proposed for rendezvous guidance. The PN algorithm is evaluated with six-degree-of-freedom (6 DOF) aircraft dynamics [8] and with external disturbances.

II. Proportional Navigation

Figure 1 defines the aerial refueling geometry. The line-of-sight angle Φ from the target location to the receiver is defined positive counterclockwise from the east and $-\pi \leq \Phi \leq \pi$. Aircraft heading ψ is defined positive clockwise from the north and $-\pi \leq \psi \leq \pi$. All positions and velocities are defined in the north and east directions in the flat-Earth plane. Aircraft heading is defined as the direction in which the nose of the aircraft is pointing, not the ground-track direction.

Lu [6] describes the course with which the interceptor will approach the target based on the proportional gain value in the PN guidance law, shown in Eq. (1):

$$\dot{\psi}_{\mathrm{com}} = -\lambda_0 \dot{\phi} \tag{1}$$

For $\lambda_0 > 2$, the intercept trajectory will end in a direct collision course with the target and $\dot{\phi} \to 0$. An assumption is made in the formulation of the proportional guidance law that the velocity of the target location is zero. As such, Eq. (2) is used in the guidance law for the rate of change of the line-of-sight angle:

$$\dot{\phi} = \frac{(\dot{y}_R)(x_R - x_{\mathrm{RZ}}) - (\dot{x}_R)(y_R - y_{\mathrm{RZ}})}{(x_R - x_{\mathrm{RZ}})^2 + (y_R - y_{\mathrm{RZ}})^2} \tag{2}$$

The proportional gain in Eq. (1) must ensure a direct collision course with the target (i.e., $\lambda_0 > 2$) and it must satisfy rendezvous

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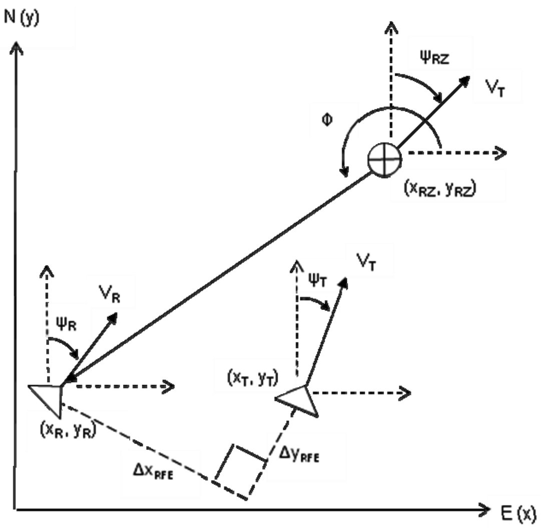

Fig. 1 Aerial refueling rendezvous geometry in a horizontal flat-Earth plane.

constraints. At rendezvous, the receiver heading and velocity must equal the heading and velocity of the tanker. The heading constraint is given by

$$\Psi_f = \psi_{\mathrm{RZ}} \tag{3}$$

where ψ_{RZ} is the target heading at the predicted point of rendezvous. Lu et al. [7] show that when perfect tracking is assumed and the guidance law is integrated, the proportional gain that ensures Eq. (3) also satisfies Eq. (4). Hence, there is a unique proportional gain that will satisfy the rendezvous heading constraint:

$$\lambda_0 = \frac{-(\Psi_f - \psi_0)}{\Phi_f - \phi_0} \tag{4}$$

Lu [6] establishes, assuming perfect tracking of the guidance command, that the guidance law will result in the condition shown in Eq. (5):

$$\Phi_f = -\frac{\pi}{2} - \Psi_f \tag{5}$$

The proportional navigation is not engaged until the proportional gain is greater than 2 to ensure a direct collision course with the target. There exists some range of headings at any given line-of-sight angle that will result in the proportional gain being greater than 2. If the receiver heading is not in that range, it must be reoriented to achieve the necessary gain condition. The turn-rate command shown in Eq. (6), analogous to the bank-angle command suggested in [7], guarantees an increase in the proportional gain and is used until Eq. (4) is greater than 2:

$$\dot{\psi}_{\mathrm{com}} = \dot{\psi}_{\max} \operatorname{sgn}(\Phi_f - \phi) \tag{6}$$

III. Adaptation of Proportional Gain

The guidance law developed in Sec. II will achieve the final heading constraint with the assumption of perfect tracking and no external disturbances. However, when subjected to tracking errors, winds, modeling uncertainties, etc., the guidance law with the gain from Eq. (4) may not result in a trajectory satisfying Eq. (3). Consequently, the gain must be updated to account for any guidance errors. The update is considered adaptive in the sense that it is a function of time and is similar to that shown in [7]:

$$\frac{\mathrm{d}\lambda}{\mathrm{d}t} = \frac{\kappa}{s}\left(\lambda + \frac{\Delta\psi}{\Delta\phi}\right)\frac{\mathrm{d}s}{\mathrm{d}t} \tag{7}$$

where κ is an arbitrary tunable gain that scales the rate of adaptation. The assumption made for Eq. (2) is again made and the resulting adaptation is shown in Eq. (8):

$$\dot{\lambda} = \frac{\kappa}{s^2}\left(\lambda + \frac{\Delta\psi}{\Delta\phi}\right)((x_R - x_{\mathrm{RZ}})(\dot{x}_R) + (y_R - y_{\mathrm{RZ}})(\dot{y}_R)) \tag{8}$$

IV. Rendezvous Point Estimator

To prevent prolonged tail chases by the receiver aircraft, the PN guidance system commands the receiver to a rendezvous location by predicting a rendezvous target point based on current state measurements each time the guidance system updates. Accordingly, the target location must eventually coincide with the tanker. The receiver is assumed to have no knowledge of the tanker's planned flight path. A new axis system is defined so that the flat-Earth axes are rotated by the tanker's heading and is referred to as the rotated flat-Earth frame. Kinematics based on the current speed and turn rate of the tanker are used to calculate the target location, assuming constant speed and constant turn rate. The instantaneous estimate of the time to rendezvous is given by

$$\hat{t} = \frac{R_{\mathrm{TR}}(1 + |\Delta\psi|)}{V_{\mathrm{Rg}}} - \left\{\frac{|\Delta y_{\mathrm{RFE}}|}{V_{\mathrm{Rg}}}\left(\frac{-(\Delta\psi - \frac{\pi}{2})^3}{(\pi/2)^3}\right)\right\} \tag{9}$$

$$R_{\mathrm{TR}} = \sqrt{(x_R - x_T)^2 + (y_R - y_T)^2} \tag{10}$$

$$\Delta\psi = \psi_R - \psi_T \tag{11}$$

The first term on the right-hand side of Eq. (9) computes an estimate of the time to rendezvous based on the distance between the tanker and receiver and the difference in their headings. The second term on the right-hand side of Eq. (9) is a factor that ensures that $\hat{t}$ will go to zero and that the target location will eventually coincide with the tanker. The range component Δy_{RFE} will always be less than or equal to R_{TR} and will only equal R_{TR} when the receiver is directly behind the tanker.

V. Velocity Controller

The receiver must arrive at the rendezvous location with the same velocity as the tanker. This is achieved by accelerating the vehicle throughout its trajectory to meet the tanker at the rendezvous location at the same time and with zero acceleration:

$$V_f = V_T \tag{12}$$

$$A_f = 0 \tag{13}$$

The following formulation is a novel approach at coupling a velocity controller to the PN guidance and rendezvous point estimator to achieve the terminal velocity constraint. Ochi and Kominami [9] propose a velocity controller of the form

$$a = -k\frac{(V_T - V_R)^2}{1.5R_{\mathrm{TR}}} \tag{14}$$

Equation (14) is based on the kinematics equation for constant deceleration, $a = -\frac{1}{2}(V_c^2/s)$, that results in relative speed V_c and range s going to zero at the same time. Using Eq. (14) as a baseline, a controller is developed that is valid for any relative velocity or position and is robust to tanker maneuvers such as turns or accelerations.

Distance and orientation can be combined to produce a time-to-rendezvous estimate for both aircraft. These two terms can then be compared to get a relative sense of how far ahead or behind the receiver is from the tanker. Equation (15) is the time to rendezvous

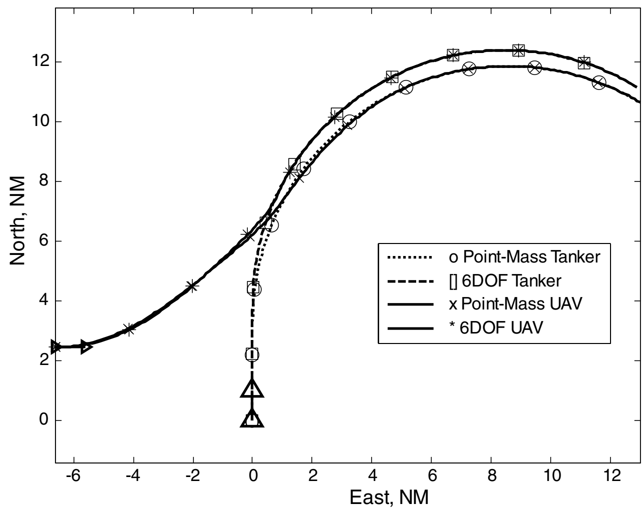

Fig. 2 Overhead view of point-mass and 6 DOF trajectories.

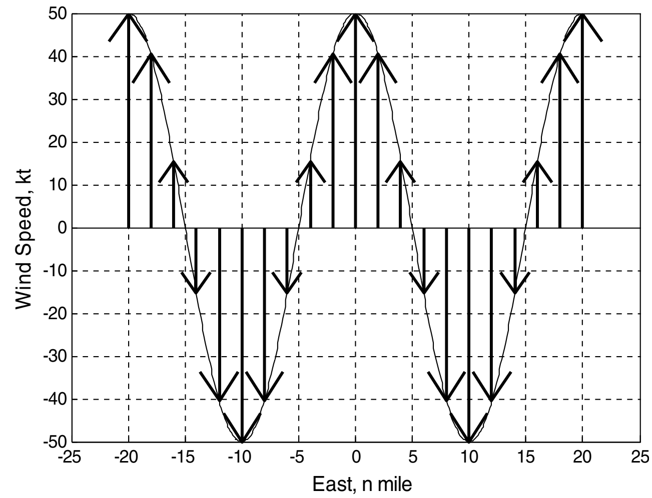

Fig. 3 Wind profile.

for the receiver:

$$t_R = \frac{R_{R\mathrm{RZ}}}{V_{\mathrm{Rg}}}(1 + |\Delta\psi|) \tag{15}$$

$$\Delta\psi = \psi_R - \psi_{\mathrm{RZ}} \tag{16}$$

$$R_{R\mathrm{RZ}} = \sqrt{(x_R - x_{\mathrm{RZ}})^2 + (y_R - y_{\mathrm{RZ}})^2} \tag{17}$$

Similarly, the time to rendezvous for the tanker is given by

$$t_T = \frac{R_{T\mathrm{RZ}}}{V_{\mathrm{Tg}}}(1 + |\Delta\psi|) \tag{18}$$

$$\Delta\psi = \psi_T - \psi_{\mathrm{RZ}} \tag{19}$$

$$R_{T\mathrm{RZ}} = \sqrt{(x_T - x_{\mathrm{RZ}})^2 + (y_T - y_{\mathrm{RZ}})^2} \tag{20}$$

The time-to-rendezvous estimates in Eqs. (15) and (18) are used to scale the tanker velocity to create a target velocity. The signum function is used to determine whether the command should be an acceleration or deceleration command. K is an arbitrary tunable gain that scales the rate of acceleration. Equation (21) shows the proposed velocity controller:

$$a = -k\frac{(V_R - \sqrt{[(t_R+1)/(t_T+1)]}V_T)^2}{R_{R\mathrm{RZ}}}\,\mathrm{sgn}\left(V_R - \sqrt{\frac{t_R+1}{t_T+1}}V_T\right) \tag{21}$$

The target velocity and acceleration command are subject to limits set in the algorithm. Equation (21) will result in the closing velocity and range to target going to zero at the same time by reverting back to the constant-deceleration kinematics equation form. Assuming the following, $t_R = t_T$, $V_R > V_T$, $\phi \to \Phi_f$, $(V_R - V_T) = V_c$, and $k = \frac{1}{2}$, Eq. (21) becomes $a = -\frac{1}{2}(V_c^2/s)$.

Table 1 UAV limits

Turn Rate	$-2 \le \dot{\psi} \le 2$	deg/s
Acceleration	$-2 \le \dot{V}_t \le 2$	$\mathrm{ft/s^2}$
Airspeed	$600 \le V_t \le 800$	ft/s

VI. Simulation

A. Six-Degree-of-Freedom Simulation

The preceding guidance system was developed assuming point-mass dynamics and perfect tracking of commands. It is tested using a 6 DOF UAV model [8], along with a high-fidelity 6 DOF KC-135 tanker model, to represent an actual GPS-guided rendezvous. Perfect communication is assumed and the dynamics models and communication are integrated and transmitted, respectively, at 80 Hz.

Figure 2 shows simulation results of a rendezvous with the tanker starting at the origin and flying north, the UAV starting approximately 6.5 n mile west–northwest of the tanker and flying east, and the tanker turning after 30 s of simulation time. The UAV starts at an airspeed of 750 ft/s, and the tanker is flying at 670 ft/s. Triangles represent the initial position and direction of both aircraft. Differences in trajectories can clearly be seen between the point-mass representation with perfect tracking and the 6 DOF representation. However, the PN guidance system responds successfully to UAV dynamics, imperfect UAV command tracking, and tanker dynamics and commands a successful rendezvous. The guidance system is also constrained by the command saturation limits shown in Table 1.

B. Simulation with Wind

A rendezvous guidance system must be able to account for any external disturbances. A wind model is used that will exercise the PN guidance system's ability to withstand these disturbances by using steady winds for which the strength is a function of east position. The tanker and receiver will be subjected to varying and unequal wind strengths throughout their respective flights. Figure 3 shows the wind profile used; airstreams will blow in the north and south directions. Because both the tanker and receiver are turning throughout their respective trajectories, the wind will also act in varying relative directions on each aircraft.

Multiple initial positions and orientations were used to demonstrate the PN guidance's ability to achieve a rendezvous in winds independent of the starting location and direction and subject to the limits in Table 1. Table 2 lists the UAV initial conditions for the rendezvous cases shown here. The tanker starts at the origin flying north at 670 ft/s for all cases. Rendezvous is declared in the

Table 2 Simulation initial conditions, UAV

Trajectory	North, ft	East, ft	Heading, deg	Airspeed, ft/s
■	15,000	−80,000	90	750
◆	100,000	−60,000	135	750
●	180,000	180,000	−110	750

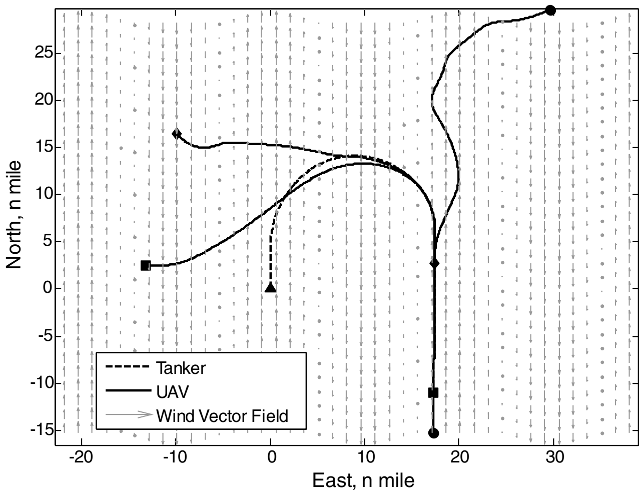

Fig. 4 Rendezvous trajectories with wind.

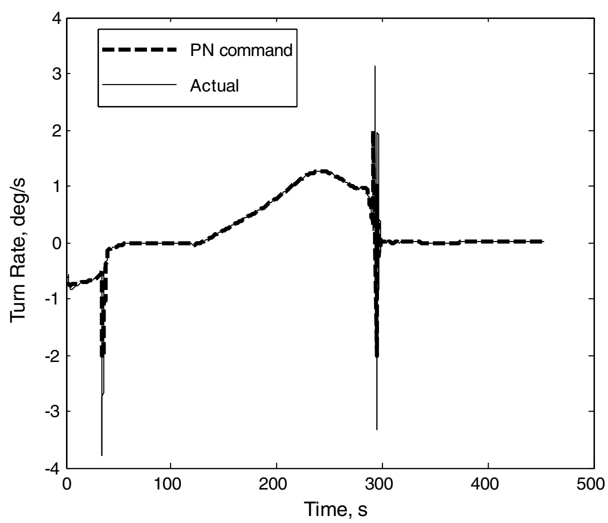

Fig. 5 Turn-rate control history for the square trajectory.

simulation when the closure rate is less than 1 ft/s for more than 20 s. Multiple runs of the simulation have shown that the final trajectory flown by the receiver is very sensitive to small changes in initial position and orientation. Sensitivities are caused by the coupling between the adaptive PN, the rendezvous point estimator, and the velocity controller. Wind also causes some sensitivity. Figure 4 shows three rendezvous trajectories commanded by the PN at different starting positions listed in Table 2 with the wind profile shown in Fig. 3.

Figures 5 and 6 show guidance control histories for the trajectory denoted with a square in Fig. 4. The control command limits are apparent in each figure. Significant transient commands are noticeable after 30 s, when the tanker begins its turn, and at approximately 295 s, when the receiver converges onto the tanker's flight path and continues to close the remaining distance.

At simulation termination for all three trajectories, the receiver is within 19 feet of the tanker, 0.01 deg of the tanker's heading, and 0.7 ft/s of the tanker's airspeed. The trajectory denoted with a square took 453 s for a rendezvous, the diamond took 316 s, and the circle took 496 s.

VII. Conclusions

A proportional navigation guidance system augmented with adaptive terminal guidance and a coupled velocity control loop is effective at executing a successful midair constant-altitude

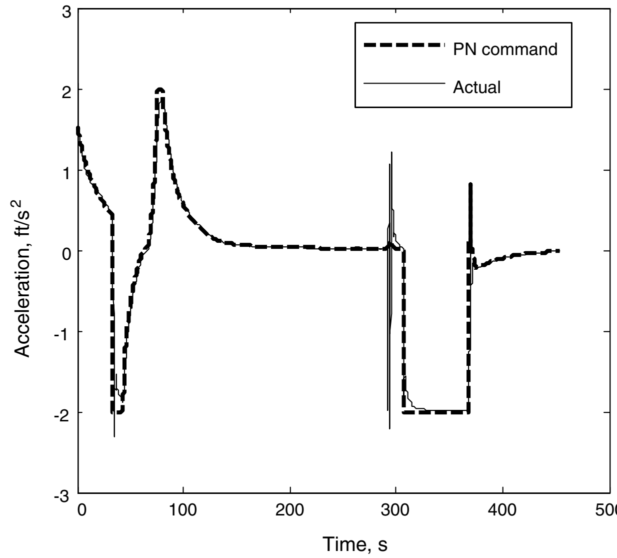

Fig. 6 Acceleration control history for the square trajectory.

rendezvous between an unmanned receiver aircraft and tanker aircraft for the purpose of automated aerial refueling. This guidance system is robust to six-degree-of-freedom aircraft dynamics, unknown tanker maneuvers, saturation limits of guidance commands, and varying winds. Proportional navigation is a simple guidance method to implement that is not hindered by convoluted mathematics or iterative loops used in other methods such as route planning and dynamic optimization. When combined with terminal guidance and a coupled velocity controller, proportional navigation guidance effectively flies a receiver aircraft within operational limits to a rendezvous with a tanker, where it is subject to terminal heading and speed constraints. This guidance method is also not limited to two-dimensional trajectories; it can be expanded to varying altitudes and used to execute other forms of intercepts and rejoins.

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